Optimal Paths Through Downbursts

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The control of an aircraft flying through a downburst is formulated as a dynamic optimization problem with a minimum altitude constraint and two different performance measures. Takeoff flight is mainly considered while landing flight is discussed. Paths are determined through windshears and downdrafts that 1) maximize a combination of airspeed and altitude just after penetration and 2) minimize deviation from the intended flight path. The first may be thought of as a "survival" measure and the second as a "performance-maintaining" measure. For mild to moderate downbursts, the performance strategy maintains altitude at the expense of airspeed loss, whereas the survival strategy descends the aircraft to the minimum altitude to obtain more airspeed. For severe downburst, the two optimal paths are similar, with the aircraft spending most of the time at the minimum altitude. In other words, the aircraft has to descend in a severe downburst to survive. Since the intensity of a downburst is difficult to assess, the survival flight-path strategy is recommended when the pilot believes that he has entered a downburst. This strategy involves immediate application of maximum thrust, along with rapid descent to the minimum safe altitude. This increases airspeed and places the aircraft in a region of lower downdraft velocity, and hence enhances the survivability.

Introduction

A SERIES of windshear-related aircraft accidents has attracted the attention of many engineers and meteorologists. In the mid-1970s, Fujita[1] identified a phenomenon that he called a "downburst." Downbursts produce the most threatening types of windshears and are responsible for most of the windshear-related accidents.[2] Figure 1 shows an aircraft encountering a downburst on takeoff. Research has been conducted for some time on different aspects of such encounters. Pilot awareness and training are the most effective solutions at the present time. The question is, what strategy should an aircraft follow in a downburst?

Miele et al.[3,4] approached the takeoff flight problem via nonlinear optimization and studied the optimal trajectories of an aircraft through windshears. Eight performance indices were considered with various boundary conditions.[3] Similar studies were also made by Psiaki and Stengel.[5,6] Massive efforts were committed to calculate optimal trajectories.[6] In all of these studies, performance indices were chosen to penalize deviations from the nominal flight path.

If maintaining the nominal path as closely as possible without stall in the specified interval is defined as performance strategy, and seeking the longest possible stay in the air is defined as survival strategy, the aforementioned studies can be explained as emphasizing performance.

This paper treats the problem by studying and comparing the two most important concerns in a downburst encounter: survival and performance. Nonlinear optimal control problems with a minimum altitude constraint are formulated to achieve the best survival and the best performance, respectively. The two optimal paths are different. The survival strategy descends to and maintains the minimum altitude in order to gain airspeed and to avoid large downdrafts. The performance strategy, on the other hand, keeps a certain climb rate at the expense of airspeed loss. For severe downbursts, the two optimal trajectories are similar and are both descending paths.

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Point-Mass Equations of Motion of an Aircraft

The point-mass model of B-727 aircraft developed by Miele et al. is adopted here.[3] Details of the aerodynamic data are contained in Appendix A.

As discovered by Miele et al., a reference frame moving with the local air mass is convenient in expressing the wind components explicitly. Figure 2 shows the thrust, lift, drag, and the D'Alembert forces of the wind components acting on an aircraft in such an air mass reference frame. The equations of motion can be written as follows:

$$\dot{x} = V\cos\gamma + W_x(x,h) \tag{1}$$

$$\dot{h} = V\sin\gamma + W_h(x,h) \tag{2}$$

$$m\dot{V} = T\cos(\alpha+\delta) - D - mg\sin\gamma - m\dot{W}_x\cos\gamma - m\dot{W}_h\sin\gamma \tag{3}$$

$$mV\dot{\gamma} = T\sin(\alpha+\delta) + L - mg\cos\gamma + m\dot{W}_x\sin\gamma - m\dot{W}_h\cos\gamma \tag{4}$$

where x and h are the horizontal and vertical coordinates of the aircraft, V is the airspeed, γ is the flight-path angle relative to

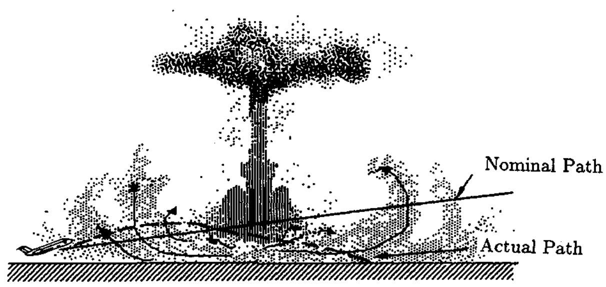

Fig. 1 Aircraft encounters a downburst on takeoff.

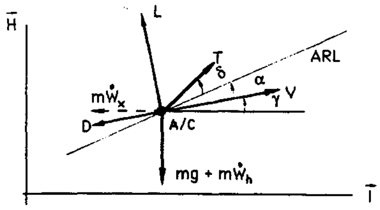

Fig. 2 Two-dimensional vertical plane.

the air mass, α is the angle of attack, δ is the thrust inclination, L and D are the lift and drag forces, T is the thrust force, and W_x and W_h are the horizontal and vertical wind components. There are four states: x, h, V, γ, and one control: α.

Downburst

Optimization poses stringent requirements on the selection of a downburst model. It should describe the downburst phenomenon adequately and yet should not cause excessive computation. In this research, Ivan's ring-vortex downburst model[7] is simplified to meet these needs.[8] The wind components at a certain point produced by this model depend on both the horizontal position and the altitude (see Appendix B). This simplified model represents a great saving in computation over the original model.

In the studies that follow, two concentric vortex rings are used to model downbursts. Two sets of downburst parameters are chosen. One represents a moderate downburst and the other a severe downburst. Table 1 contains the parameters.

Key Quantity

A tailwind-increasing windshear causes an aircraft to lose airspeed, therefore lift. A downdraft velocity directly reduces altitude. The combined effect can cause the aircraft to hit the ground. In this connection, the two most important states are the altitude and the airspeed.

Airspeed can be increased quickly by diving or decreased quickly by a rapid climb; the sum of potential energy plus kinetic energy stays nearly constant in rapid climbs and dives. It is the energy relative to the air mass that is the key quantity. The critical state of an aircraft occurs when airspeed is close to stall speed and altitude is zero.

This energy is defined as the "pseudoenergy" E:

$$E \triangleq h + (V^2/2g)$$

In this definition, the altitude h is measured relative to the ground, and the airspeed V is the speed of the aircraft with respect to the moving air mass. This explains why it is called "pseudoenergy." It is a measure of the capability of an aircraft to penetrate or survive a downburst.

Problem Formulation

It is natural to desire to keep the altitude as close as possible to the intended flight path. Previous investigators have defined their performance indices this way.[3-6] This "performance-maintaining" problem can also be expressed as a nonlinear optimization problem with the following cost function:

$$\min \max \quad |\dot{h} - \dot{h}_0|$$

where

$$\dot{h} = \text{climb rate}$$

$$\dot{h}_0 = \text{intended climb rate}$$

Climb rate is used here since altitude is a direct concern, and the survival strategy that follows can be easily compared. Minimizing the maximum altitude deviation achieves nearly the same result, but is not as numerically effective.

Table 1 Downburst parameters

Moderate		Severe		
Name	Value	Name	Value	Unit
Γ_1	200,000	Γ_1	400,000	ft²/s
R_1	5,500	R_1	5,000	ft
H_1	2,000	H_1	2,000	ft
R_{c1}	500	R_{c1}	500	ft
Γ_2	120,000	Γ_2	280,000	ft²/s
R_2	4,000	R_2	3,500	ft
H_2	2,500	H_2	2,000	ft
R_{c2}	500	R_{c2}	300	ft

Johnson[9] pointed out that this cost function is *equivalent to*

$$\min \; I = \lim_{n\to\infty} \int_0^{t_f} (\dot{h} - \dot{h}_0)^n \, dt$$

and can be *approximated by*

$$\min \quad \tilde{I} = \int_0^{t_f} (\dot{h} - \dot{h}_0)^q \, dt$$

where q is a large even integer.

The optimal solution gives the best possible performance in a downburst encounter in terms of maintaining climb rate. However, the most important concern in such a case should be survival, and the two considerations produce different results for mild to moderate downbursts, as we shall show.

As discussed before, higher pseudoenergy E represents the potential of an aircraft to survive longer. Therefore, we should like to maximize $\dot{E}$ at every instant of time. To this end, the "survival" problem is formulated to maximize the final pseudoenergy:

$$\max_{\alpha} \; E(t_f)$$

Preliminary results indicate that it is more effective to gain pseudoenergy by an increased airspeed that is accomplished by a decreased altitude. Therefore, an altitude inequality constraint has to be imposed to obtain practical solutions.

$$h \geq h_{\min}$$

This altitude constraint is used for both the performance and the survival formulations. It constitutes a second-order state inequality constraint in a time-varying wind field. A slack variable method is employed to convert this constraint into a path equality constraint.[10] Specifically, auxiliary variables y_1, y_2, and u are introduced as follows:

$$h - h_{\min} = \frac{1}{2} y_1^2$$

Double differentiations of this equation lead to the following:

$$V \sin\gamma + W_h(x,h) = y_1 y_2$$

$$T \sin(\alpha + \delta + \gamma) - D \sin\gamma + L \cos\gamma = m(g + y_2^2 + y_1 u)$$

where

$$\dot{y}_1 = y_2$$

$$\dot{y}_2 = u$$

To summarize, we have formulated two cost functionals from considerations of performance and survival, respectively. The cost functional for the "performance-maintaining'' problem is

$$\min_{\alpha, u} \quad I = \int_0^{t_f} (\dot{h} - \dot{h}_0)^q \, \mathrm{d}t \tag{5}$$

The cost functional for the survival problem is

$$\max_{\alpha, u} \ E(t_f) = h(t_f) + \left[V^2(t_f)/2g\right]$$

Both of these two problems are subject to

$$\dot{x} = V \cos\gamma + W_x(x, h)$$

$$m\dot{V} = T \cos(\alpha + \delta) - D - mg \sin\gamma - m\dot{W}_x \cos\gamma - m\dot{W}_h \sin\gamma$$

$$\dot{y}_1 = y_2$$

$$\dot{y}_2 = u$$

where

$$h = h_{\min} - \frac{1}{2} y_1^2$$

$$\sin\gamma = \frac{1}{V}(y_1 y_2 - W_h)$$

and the initial conditions:

$$x(0) = 0$$

$$V(0) = 276.8 \text{ ft/s}$$

$$y_1(0) = \sqrt{2\left[h(0) - h_{\min}\right]}$$

$$y_2(0) = \frac{V(0) \sin\gamma(0) + W_x\left[x(0), h(0)\right]}{\sqrt{2\left[h(0) - h_{\min}\right]}}$$

The path equality constraint is

$$T \sin(\alpha + \delta + \gamma) - D \sin\gamma + L \cos\gamma = m(g + y_2^2 + y_1 u)$$

Terminal constraints will be imposed in particular examples that follow.

Numerical Methods

Miele's sequential gradient restoration algorithm[11] was used to solve the preceding problems numerically. Also, a generalized gradient algorithm was developed by the authors to accomplish the same task.[8]

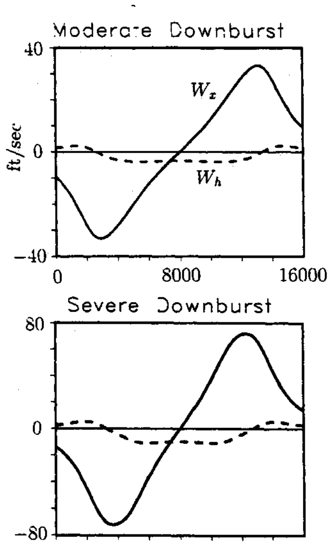

Fig. 3 Downburst profiles at $h = 300$.

Performance Takeoff Flight

Performance strategies are considered here in a moderate downburst. Figure 3 shows the wind profiles of a moderate downburst at an altitude of $h = 300$ ft. Also shown are the wind profiles of a severe downburst, which will be used later.

Since the thrust is close to maximum at takeoff, and it takes just a short time to spool the engine to maximum once a downburst is detected, maximum thrust is used throughout the following examples. We assume that the aircraft detects the downburst at $h_0 = 300$ ft. The final time is taken to be $t_f = 60$ s so that the aircraft is through the downburst at the end of the interval. The integer power $q = 6$ is employed in Eq. (5).

The aircraft is assumed to encounter the downburst at the point where the increasing headwind turns into an increasing tailwind. This corresponds to $X_1 = 5000$ ft and $X_2 = 5000$ ft (refer to Appendix B and Fig. 3). No terminal constraints are imposed. Optimal paths are shown by solid lines in Fig. 4.

The climb rate during the unfavorable wind region is roughly constant and lower than the nominal value in a windless condition. The nominal climb rate is gradually restored toward the end of the encounter. The effect of varying wind is clearly reflected in the airspeed. The airspeed decreases gradually during the unfavorable region and is restored to close to its initial value at the end of the encounter. Therefore, altitude is maintained at the expense of airspeed loss during a performance takeoff.

The absolute flight-path angle γ_e reduces to a roughly constant value during the unfavorable region and increases to a higher value toward the end of the encounter. The relative flight-path angle γ shows a similar trend with some differences caused by the downdraft. In particular, it shows a gradual increase during the unfavorable wind region due to the gradual increase of the downdraft. This means that the aircraft has to increase its climb angle with respect to air mass in an increasing downdraft in order to maintain a constant inertial climb rate.

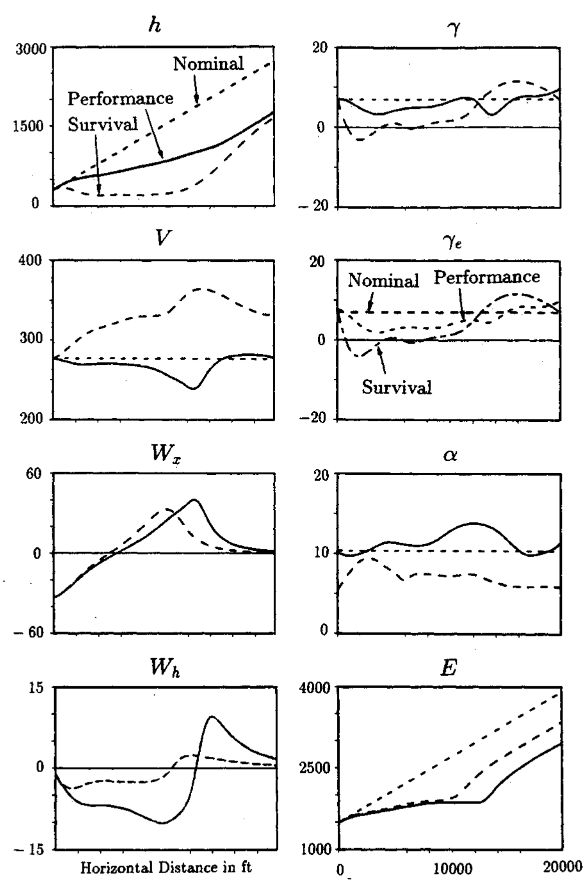

Fig. 4 Comparison: moderate downburst.

The angle of attack shows an initial reduction, which reduces the climb rate, followed by a gradual increase to attain the climb rate, and ending with a decreasing trend. The pseudoenergy shows a small but positive rate during the unfavorable wind region and a large rate during the decreasing tailwind and updraft at the end. However, the net effect of a downburst is to cause an aircraft to lose energy.

In short, altitude is maintained at the expense of airspeed loss. A constant but lower climb rate is recommended in a mild to moderate downburst encounter. This would mean reducing climb rate at the beginning. The climb rate to be followed depends on the specific wind situation and the aircraft state.

It is desirable to restore the initial airspeed and flight-path angle after a successful penetration has been made. An example problem is solved with terminal constraints $V_f = V_0$ and $\gamma_f = \gamma_0$. The paths are almost the same as before with minor differences toward the end of the interval. This is expected since the final airspeed and flight-path angle in the previous example are already close to the initial values.

Examples with only the final airspeed constraint or the final flight-path angle constraint are also studied. It is not surprising that similar results are found. The final flight-path angle constraint affects the angle-of-attack history only near the final time. The terminal airspeed constraint directly influences the final altitude. Specifically, a higher final airspeed corresponds to a lower final altitude and vice versa. The pseudoenergy is almost the same. The conclusion is that the terminal airspeed constraint affects the distribution of the energy between altitude and airspeed. These observations demonstrate that the pseudoenergy is the key quantity.

Examples using the ring-vortex downburst model of different intensities and/or different encounter locations are also studied. The results are similar. In particular, the climb rate during the unfavorable wind region is roughly constant and depends on the intensities.

Survival Takeoff Flight

Survival becomes the first concern in severe downbursts. The problem of maximizing the final pseudoenergy has been formulated previously to enhance survivability.

A severe downburst is chosen here. The maximum horizontal wind velocity is about 80 ft/s. Bear in mind that downbursts can produce winds as large as 225 ft/s! Figure 3 shows the wind profiles at an altitude of 300 ft.

We assume that the aircraft encounters the downburst at an initial altitude of $h_0 = 500$ ft. The minimum altitude constraint is taken to be $h_{\min} = 200$ ft. The length of time is taken to be $t_f = 60$ s. The final airspeed and flight-path angle are required to return to their initial values: $V_f = V_0$ and $\gamma_f = \gamma_0$.

The encounter is assumed to occur a little bit before the increasing headwind changes to an increasing tailwind. This corresponds to $X_1 = X_2 = 5000$ ft in the current example. The results of optimization are shown in Fig. 5 by the dashed lines.

Since the final airspeed is constrained, the problem is equivalent to that of maximizing the final altitude. The altitude history shows a brief climb followed by a long period at the minimum altitude. When the downburst becomes favorable, the aircraft starts to climb. The increased airspeed at the beginning is due to the effect of increasing headwind. When the windshear and downburst become unfavorable, the airspeed starts to decrease. Airspeed increases after the downburst becomes favorable and reaches the initial value at the end.

The relative flight-path angle and the absolute flight-path angle show patterns that are consistent with those of the preceding airspeed and altitude. The initial reductions in both γ and γ_e correspond to the pitchdown motion. The constant arcs are at the minimum altitude. Then, the absolute flight-path angle gradually increases, while the relative flight-path angle decreases before it increases, due to the variation of downdraft.

The angle of attack exhibits a large reduction at the beginning to reduce the climb rate to zero. During the $h = h_{\min}$ arc, the angle of attack increases in a sustained way to cope with the decreasing airspeed caused by the change from headwind to tailwind coupled with downdraft. The final restoration of both the airspeed and the flight-path angle are accomplished by a lower angle of attack, using the extra energy input from the downburst. The angle of attack is generally low, and nowhere exceeds the stall limit. The pseudoenergy history is more revealing of the downburst nature. The initial higher rate of energy increase is followed quickly by a negative rate of increase.

A simple analysis using energy approximation indicates that it is more effective to increase energy by increasing airspeed. Different intensities from mild to severe and/or different encounter locations are considered with the survival strategy. The results are similar in that the aircraft descends to gain speed. Different final conditions are found to affect the terminal part of the trajectories only. During the unfavorable downburst region, the aircraft is at the minimum altitude. The airspeed may increase, stay roughly constant, or decrease, depending on the intensities of the downburst scenario.

A better survival capability is obtained if the initial airspeed is larger. For a fixed initial altitude, a lower minimum altitude is found to correspond to higher airspeed and vice versa.

The results of minimizing the transit time across a downburst and of surviving the greatest distance with zero vertical impact rate share some similarities with the maximizing energy scheme. Readers interested in these details can find them in Ref. 8. These considerations together constitute the *survival strategy*. In short, an aircraft should trade altitude for airspeed to obtain a better survival capability.

Comparisons of the Two Strategies

The two strategies discussed previously represent conflicting interests. One emphasizes performance whereas the other emphasizes survival. As a result, one has a better altitude history whereas the other has a better airspeed history. For the downburst models used in the previous sections, results of both the performance-takeoff and the survival-takeoff strategies are

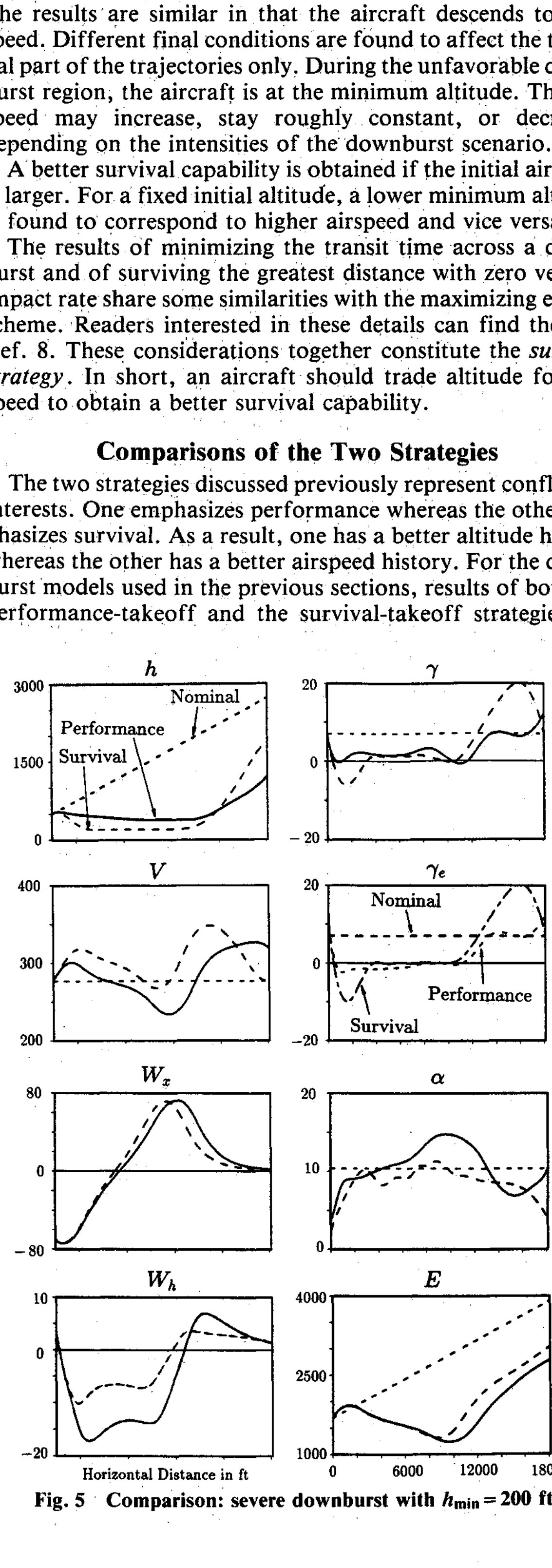

Fig. 5 Comparison: severe downburst with $h_{\min} = 200$ ft.

plotted in Figs. 4 and 5. In the comparison using the moderate downburst (see Fig. 4), the initial altitude is 300 ft, and the minimum altitude for the survival strategy is 200 ft. In the comparison using the severe downburst (see Fig. 5), the initial altitude is 500 ft, while the minimum altitude is 200 ft.

The performance strategy achieves a monotonic climbing altitude in the moderate downburst at the expense of airspeed loss. In the severe downburst, the altitude shows a slight decrease since the optimal trajectory knows that an initial climb will be penalized by a stall at a later time. In comparison, the survival strategy path always descends to a minimum safe altitude upon entering the downburst. The severities of the downbursts are reflected through speed variations. The airspeed increases in the moderate downburst, whereas it decreases in the severe one.

The two energy histories are very close to each other in the first half of the downbursts but differ in the second half due to differences in the windshears and downdrafts they experience. The survival strategy ends up with a higher final pseudoenergy as expected.

The comparison demonstrates that the performance strategy is also to descend in a severe downburst. When the severity increases, the aircraft has to descend further simply to get across. Figure 6 shows the comparison of the two strategies in the severe downburst with a higher minimum altitude constraint: $h_{\min} = 400$ ft. The minimum altitude constraint is so overwhelming that the two strategies look very much alike. Comparisons using very severe downbursts reveal the same conclusion. In other words, the basic survival concern in severe downbursts requires aircraft to descend willingly (as in survival strategy) or unwillingly (as in performance strategy).‡ Therefore, in very severe downbursts where the survival problem becomes paramount, the aircraft must descend regardless of which strategy it wishes to follow.

The final energies of the survival strategy are higher than those of the performance strategy in all these comparison plots. Higher airspeed helps to increase energy. This is also true because the optimization knows the global structure of the downburst model; thus the aircraft following the survival strategy experiences wind components of slightly smaller intensities. On the other hand, actual downbursts exhibit some common features. The downbursts occur close to the ground, and as the altitude drops, the downdraft reduces and the horizontal wind component does not change very much if it does not decrease. Therefore, it is usually beneficial to fly at a low altitude.

If the wind components experienced by performance and survival strategies are the same, the pseudoenergy as a function of distance will be close, since the energy of an aircraft is mainly changed by thrust and the wind components and only slightly affected by angle of attack. The angle-of-attack variation does alter the distribution of the energy between altitude and airspeed. The key difference between these two strategies is in the energy distribution. In this connection, the performance strategy trades airspeed for altitude, whereas the survival scheme preserves airspeed at the expense of poor altitude behavior. The former emphasis is on a trajectory close to the intended flight path, while the latter is good for survival.

The survival strategy is insensitive to downburst severity, since the strategy is always to have the aircraft descend to and maintain a minimum safe altitude until the aircraft is essentially through. It may seem pessimistic in a mild to moderate downburst but it is still easy and safe. The difficult problem in a performance strategy is to determine what climb rate to follow. Since remote-sensing technology is not available, it is impossible to know the downburst information prior to penetration. Therefore, the survival strategy is recommended.

The minimum altitude is a subjective choice. The lower it is, the higher the final energy will be, and therefore the better the survival capability of the aircraft. On the other hand, too low a minimum altitude would not be safe because of buildings and hills.

‡The authors thank Professor Breakwell for pointing this out.

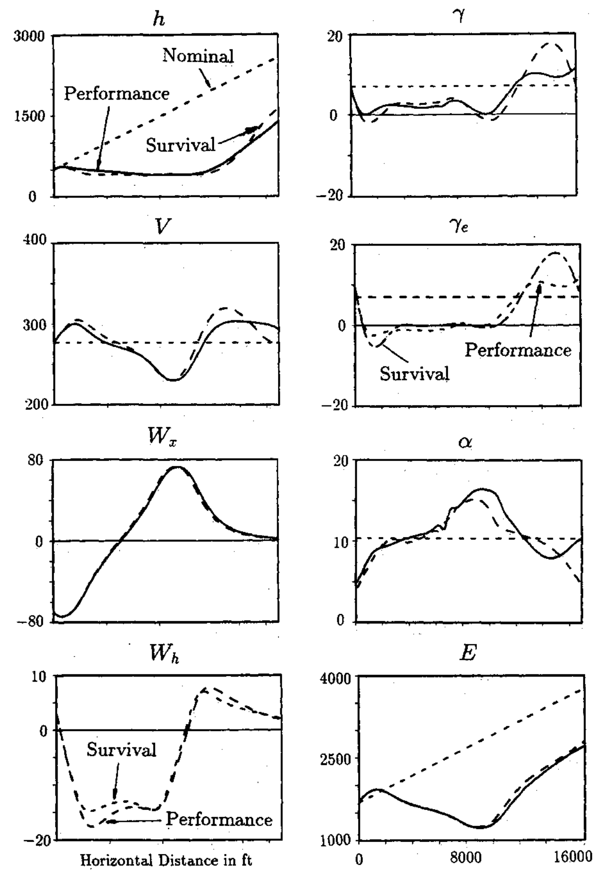

Fig. 6 Comparison: severe downburst with $h_{\min} = 400$ ft.

In an actual flight through a downburst, the hardest problem is probably to detect it and respond promptly. Then, pilots may choose to climb at a lower rate or to descend to a certain safe altitude. If the airspeed is still decreasing when close to stall, further altitude has to be traded for airspeed. In very severe downbursts, the aircraft should immediately descend to and maintain the lowest safe altitude.

Energy arguments support the use of maximum thrust. If the survival strategy is followed in downburst encounters, full thrust becomes the limiting factor.

Landing

Upon encountering a downburst in a landing flight, the pilot has two choices: to abort the landing or to proceed.

The strategy of maximizing the final energy with a minimum altitude constraint applies to abort landing since the ultimate goal is to stay in the air. The engines need some time to spool up in this case, because they are close to idle in nominal landing. This fact requires that pilots choose a lower minimum safe altitude.

The key problem in approach landing is energy fluctuation. Since the aircraft is going to come into contact with the ground, both descent rate and inertial speed should be in the right range. Therefore, nominal inertial energy should be maintained with a minimum airspeed constraint. Thrust must be varied either to add energy to or to subtract energy from the aircraft to balance the effect of the downburst. Angle of attack can then be used to distribute the energy between inertial speed and altitude.

Conclusions

Optimal aircraft takeoff trajectories in a downburst were studied with a minimum altitude constraint and two cost functions. Pseudoenergy consisting of airspeed and altitude was considered a key quantity. The performance-maintaining cost functional penalizes deviations from the intended flight path, whereas the survival strategy maximizes the final pseudoenergy just after penetration. A simplified ring-vortex downburst model was used, and different intensities and/or encounter locations were assumed. Full thrust was employed in all cases.

The performance strategy maintains a roughly constant climb rate in the unfavorable wind region at the expense of airspeed loss. The climb rate is lower than the nominal value and is higher in mild downbursts and lower in moderate downbursts. The survival strategy descends the aircraft to and maintains the minimum altitude constraint. The effect of the downburst is reflected in airspeed fluctuation. The airspeed is generally higher in mild downbursts and lower in severe downbursts. In a severe downburst and/or with a high minimum altitude constraint, the minimum altitude constraint dominates the flight so that the two strategies look very much the same; both descend to and maintain the minimum altitude.

Since the intensities of a downburst are difficult to assess prior to penetration, the survival strategy is recommended.

Appendix A: Data for the B-727 Aircraft

The model of a B-727 formulated by Miele[3] is used here, where

$$W = mg = 180{,}000$$

$$T = A_0 + A_1 V + A_2 V^2$$

$$C_D(\alpha) = B_0 + B_1\alpha + B_2\alpha^2$$

$$C_L(\alpha) = C_0 + C_1\alpha + C_2(\alpha - \alpha_1)^2$$

and where $L = qSC_L(\alpha)$, $D = qSC_D(\alpha)$, $q = \rho V^2/2$, $\rho = 0.002203$ slug ft^{-3} is the mass density of the air

$$A_0 = 44{,}560, \qquad A_1 = -23.98, \qquad A_2 = 0.01442$$

$$B_0 = 0.07351, \qquad B_1 = -0.08617, \qquad B_2 = 1.996$$

$$C_0 = 0.1667, \qquad C_1 = 6.231$$

$$C_2 = \begin{cases} 0 & \text{if } \alpha < \alpha_1 \\ -21.65 & \text{if } \alpha > \alpha_1 \end{cases}$$

$$\delta = 2\pi/180, \qquad \alpha_1 = 12\pi/180, \qquad S = 1560$$

and the units are pounds, feet, seconds, and radians.

Appendix B: Simplified Ring-Vortex Downburst Model

In the following notation, Γ represents the circulation, R represents the radius of the ring, and R_c represents the radius of the finite core. X, Y, and H are the coordinates of the primary ring center.

In two-dimensional space, the lateral position of the vortex ring center is usually assumed to be zero: $Y = 0$. The induced velocities at any point of interest (x, h) are computed through the following relations:

$$x_1 = x - X - R$$

$$x_2 = x - X + R$$

$$h_p = h - H$$

$$h_m = h + H$$

$$r_{1p} = x_1^2 + h_p^2$$

$$r_{2p} = x_2^2 + h_p^2$$

$$r_{1m} = x_1^2 + h_m^2$$

$$r_{2m} = x_2^2 + h_m^2$$

$$\zeta = 1 - e^{-(r_0/R_c^2)}$$

$$r_{xp} = \sqrt{(x-X)^2 + h_p^2 + R^2}$$

$$r_{xm} = \sqrt{(x-X)^2 + h_m^2 + R^2}$$

$$r_{hp} = \left[(x-X)^2/4 + h_p^2 + R^2\right]^{1/4}$$

$$r_{hm} = \left[(x-X)^2/4 + h_m^2 + R^2\right]^{1/4}$$

$$r_0 = \min\{r_{1p}, r_{2p}\}$$

If $r_0 < \epsilon$ (the point of interest is close to the ring filament), where $\epsilon > 0$ is a small number,

$$W_x = 0 \qquad W_h = 0$$

Else,

$$W_x = \frac{1.182\Gamma\zeta}{2\pi}\left[\frac{R}{r_{xp}}\left(\frac{h_p}{r_{2p}} - \frac{h_p}{r_{1p}}\right) - \frac{R}{r_{xm}}\left(\frac{h_m}{r_{2m}} - \frac{h_m}{r_{1m}}\right)\right]$$

$$W_h = \frac{1.576\Gamma\zeta}{2\pi}\left[\frac{R}{r_{hp}}\left(\frac{x_1}{r_{1p}^{3/4}} - \frac{x_2}{r_{2p}^{3/4}}\right) - \frac{R}{r_{hm}}\left(\frac{x_1}{r_{1m}^{3/4}} - \frac{x_2}{r_{2m}^{3/4}}\right)\right]$$

A multiple ring-vortex model can be used to provide a better description, where several of the previously developed vortex ring models are placed together. The induced velocities of such a flowfield are the vector sums of the velocities from each pair.

Acknowledgments

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